Journal of Statistical Physics, Vol. 88, Nos. 1/2, 1997

Book Review: *Turbulence: The Legacy of A. N. Kolmogorov*

Turbulence: The Legacy of A. N. Kolmogorov, Cambridge University Press, 1996, 296 pages, $29.95 (softcover).

"Never have so many—with so much—accomplished so little" might be a pithy phrase to characterize the reaction of some cynical physicists to the field of turbulence. "So many" refers to the fact that turbulence continues to hold the fascination and attention of a significant fraction of the physics research community more than 50 years after Kolmogorov's (and more than 75 years after Richardson's) seminal advances. "So little" reflects the fact that despite a truly gargantuan effort involving the best minds over the past century, the problem is by no means completely solved. "With so much" refers to the fact that the turbulence community has drawn remarkably well on the full resources of sophisticated experimentation, high-speed computing, and abstract theory.

No wonder, then, that there is no easy way for the nonspecialist to figure out answers to simple questions such as "What is known? What is not known? What is not known but should be?" Professor Uriel Frisch, among the most outstanding of all turbulence researchers, has done a great service by writing a readable book that addresses these questions.

Frisch kicks off his monograph with a 12-page introduction to some qualitative facts—illustrated by 14 figures from van Dyke's classic gem *An Album of fluid Motion* (The Parabolic Press, Stanford, 1982). The formalism begins in Chapter 2, "Symmetries and conservation laws," and here we realize that the book requires a level of mathematical (and physical) sophistication more likely to be found in non-US graduate students than in US graduate students—yet surely someone who has taken a course in fluid mechanics can follow the development. Motivated by a lovely discussion of hot wire anemometer data, Frisch next introduces (in Chapter 3) the probabilistic description, and follows (in Chapter 4) with a rather more formal development.

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Chapter, 5 discusses in some depth the experimental data on which are based two key laws of fully developed turbulence, (i) the "2/3 law" (which states that the velocity difference between two points scales as the distance between the points, $l^{2/3}$, and (ii) the energy dissipation law (which states that when the viscosity ε approaches zero, the energy dissipation does not). One purpose of Kolmogorov's classic 1941 theory is to get around the fact that it is impossible to deduce the two experimental laws from the Navier–Stokes equations. Chapter 6 is a coherent explanation of the main results of this theory, including the famous "4/5 law" (the cube of the velocity difference between two points scales as the product of εl, with an overall proportionality factor of 4/5), and the related law that the energy spectrum $E(k)$ scales as $\varepsilon^{2/3}k^{-5/3}$.

Chapter 7 recasts the Kolmogorov theory using different phenomenological language and images, introducing the Richardson energy cascade in which energy fed into the system at a large scale is "dissipated" through a hierarchy of ever smaller scales—the phenomenon that every student is perhaps first exposed to via Richardson's 1992 take-off on Jonathan Swift, "...a flea. Hath smaller fleas that on him prey; And these have smaller yet to bite 'em. And so proceed ad infinitum." This ditty has its modern counterpart in finance, for one finds on recent pages of Nature discussions of the degree to which the energy cascade of turbulence has its analog in an "information cascade": information is fed into the stock market in the form of large scale events, and this information is processed in a hierarchy of ever smaller units down to the level of the individual investor.

The 75-page Chapter 8 is devoted to a remarkably thorough discussion of the phenomenon of intermittency, presenting the experimental data as well as the β model and a coherent discussion of multifractal models. Finally, Chapter 9 summarizes, in 60 pages, some of the topics that were omitted from the more coherent course comprising the first eight chapters—including a succinct discussion of two-dimensional turbulence. Chapter 9 also contains an annotated bibliography of books and review articles, many of which approach the subject rather differently than Frisch does.

Every statistical physicist should add this book to his or her library. Instructors should consider fashioning a course in fluid dynamics and turbulence, using this book as a principal text, supplemented by other resources covering topics perforce omitted from this slim volume. The low price the Cambridge University Press has been achieving for its monographs lately ($29.95 list, in this case) might motivate some instructors of statistical mechanics to consider devoting a part of their course to this key application of statistical mechanical concepts and techniques.

Thanks to Frisch's book, it will become inexcusable that a Ph.D. student's education not include exposure to some of the key ideas of this important—and still not fully understood—topic.

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